Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 1, 1999

NUMERICAL SIMULATION OF GAS-DYNAMIC PROCESSES IN A PIPE OPEN AT ONE END

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UDC 533.6.11

The problem of sudden opening of one end of a circular pipe containing a pressurized gas is considered. A new form of the boundary condition at the open end of the pipe is proposed that takes into account the local hydrodynamic drag due to the nonlinearity of the real physical problem. The system of gas-dynamic equations is integrated by the Godunov numerical method of discontinuity decay. The procedure of numerical realization of the nonlinear boundary condition at the open end of the pipe is described in detail. Comparison of the graphs obtained in the calculations with experimental data indicates that the proposed technique is appropriate.

Introduction. The problem of discharge from a circular pipe into a medium with specified constant pressure is considered classical in the theory of unsteady flows. The main feature of the problem is the occurrence of an oscillation process that gives rise to reverse flows and leads to penetration of the ambient gas into the tube.

The problem was investigated by many authors [1–3]. What the cited papers have in common is that constant pressure equal to the pressure at infinity is specified as the boundary condition at the open end of the pipe. The remaining gas-dynamic flow parameters at the outlet are determined using the conservation laws. The solution thus obtained has the character of an undamped oscillation process, which is inconsistent with the experimental data of Levy and Potter [4] since the authors of the theoretical studies did not take into account the energy loss at the open end of the pipe due to the flow nonlinearity.

Voevodin and Safin [5] and Yaushev [6] studied problems of numerical simulation of one-dimensional gas flows in a complex pipework with allowance for the local drags introduced only at the joints of neighboring one-dimensional segments by specifying internal boundary conditions. The latter, representing the integral laws of conservation of mass, momentum, and energy, reduce to solving the problem of discontinuity decay at the jump of the cross-sectional area of the pipe [7–10]. At the ends of the one-dimensional segments, the boundary conditions are specified routinely, ignoring local drags [5, 6].

In the present paper, regularities of gas-dynamic pipe flows are investigated with allowance for the local drag at the pipe outlet. The problem is solved numerically by the Godunov method of discontinuity decay [11]. The procedure of numerical realization of the boundary condition with allowance for the drag is described in detail. It is universal and can be used successfully in other numerical methods. Results of numerous calculations are presented in graphical form, compared with experimental results, and confirm the effectiveness and expediency of the proposed procedure.

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State Scientific Engineering Center of Systems of Control and Emergency Response, State Atomic Energy Committee of Ukraine, Kiev 252011. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 1, pp. 74–79, January–February, 1999. Original article submitted December 10, 1996; revision May 22, 1997.

1. Formulation of the Problem. Let a gas at pressure $p = p_0$ and density $\rho = \rho_0$ be contained in a cylindrical pipe closed by diaphragms. At the time $t = 0$, the right diaphragm is removed, and at $t > 0$, the gas begins to discharge into the ambient medium with pressure $p = p_\infty$. It is required to describe the gas flow originating in the pipe.

Within the framework of the model of an inviscid non-heat-conducting gas, the equations describing the unsteady behavior of the pipe flow have the form [11]

$$\frac{\partial \mathbf{a}}{\partial t} + \frac{\partial \mathbf{b}}{\partial x} = 0; \tag{1.1}$$

$$\mathbf{a} = \left\| \begin{matrix} \rho \\ \rho u \\ \rho(e + u^2/2) \end{matrix} \right\|, \qquad \mathbf{b} = \left\| \begin{matrix} \rho u \\ \rho u^2 + p \\ \rho u(e + u^2/2) + pu \end{matrix} \right\|. \tag{1.2}$$

Here e is the internal energy of a unit mass of the gas and u is the velocity in the x direction. The system of differential equations (1.1) and (1.2) is closed by the equations of state for an ideal gas:

$$e = \frac{p}{(æ - 1)\rho}, \tag{1.3}$$

where æ is the adiabatic exponent.

The nonpenetration condition $x = 0$ is imposed as the boundary condition at the left boundary $u = 0$. As regards the boundary $x = L$, according to the modified Bernoulli equation, the difference between the "true" pressure at the outlet section and the external pressure $p = p_\infty$ is equal to the drag. With allowance for the direction of the flow, this relation is written as

$$p - p_\infty = \xi\rho\frac{|u|u}{2} \qquad \text{for} \qquad x = L, \tag{1.4}$$

where ξ is a constant [12] that takes into account local drags of this or that nature.

2. Procedure of Numerical Solution and Method of Realization of Boundary Conditions. To solve the formulated initial boundary-value problem, we use the Godunov method of discontinuity decay [11]. The traditional notation of the method is adopted. The problem of discontinuity decay is an elementary operation for this method. The formulas required for internal nodes of the difference grid are given in [11]. We consider the method of realizing the boundary conditions.

The condition of nonpenetration at the left end of the pipe $x = 0$ reduces to solving the problem of discontinuity decay for symmetrical initial data. The end $x = L$ is identified with a node of the grid $j = J$. The boundary condition at the right end $x = L$ in the adopted notation is written as

$$P_J - p_\infty - \xi(R_1)_J\frac{|U_J|U_J}{2} = 0. \tag{2.1}$$

The relation for the density R_1 to the left of the contact discontinuity (in the zone adjacent to the left wave from the right) is somewhat different from the one in [11]:

$$R_1(P) = \begin{cases} (\rho_1 a_1)[a_1 - \rho_1 f(P, p_1, \rho_1)]^{-1} & \text{if} \quad P > p_1; \\ (æP)[c_1 + (æ - 1)f(P, p_1, \rho_1)/2]^{-2} & \text{if} \quad P < p_1, \end{cases} \tag{2.2}$$

Here p_1, ρ_1, and c_1 are the pressure, density, and velocity of sound in the extreme right cell of the difference grid and $a_1 = a_1(P, p_1, \rho_1)$ is the mass velocity. To simplify the consideration, we omit the subscripts, except for ∞, and designate $f(P) = f(P, p_1, \rho_1) = (P - p_1)/a_1(P, p_1, \rho_1) = (P - p)/a(P)$. The expression for the mass velocity $a(P)$ on the left (shock or rarefaction) wave is given in [11].

Supplementing condition (2.1) by the condition on the left wave, we obtain the following system of transcendental equations that define P and U:

$$P - p_\infty - \xi R(P)\frac{|U|U}{2} = 0; \tag{2.3}$$

$$U - u + \frac{P - p}{a(P)} = 0. \tag{2.4}$$

Thus, the problem of numerical realization of the boundary condition at the open end of the pipe reduces to the problem of discontinuity decay with the dynamic compatibility condition (2.4) satisfied on the left wave and condition (2.3) satisfied on the right wave.

System (2.3), (2.4) is a system of the form

$$\mathbf{F}(\mathbf{x}) = \left\| \begin{matrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{matrix} \right\| = 0, \qquad \mathbf{x} = \left\| \begin{matrix} x_1 \\ x_2 \end{matrix} \right\| = \left\| \begin{matrix} P \\ U \end{matrix} \right\|. \tag{2.5}$$

We find the Jacobi matrix of system (2.5):

$$W(\mathbf{x}) = \mathbf{F}'(\mathbf{x}) = \left\| \frac{\partial f_m}{\partial x_n} \right\| = \|F'_{mn}\|, \qquad \frac{\partial f_1}{\partial x_1} = 1 - \xi \frac{|U|U}{2} R'(P),$$

$$\frac{\partial f_1}{\partial x_2} = -\xi |U| R(P), \qquad \frac{\partial f_2}{\partial x_1} = f'(P), \qquad \frac{\partial f_2}{\partial x_2} = 1.$$

We consider the possible situations:

(a) $P > p$ (a shock wave propagates to the left). Using the expressions for $a(P)$ and $R(P)$, we obtain

$$a'(P) = \frac{(æ + 1)\rho}{4a(P)}, \qquad R'(P) = \frac{\rho^2 [a(P) f'(P) - a'(P) f(P)]}{[a(P) - \rho f(P)]^2};$$

(b) $P < p$ (a rarefaction wave propagates to the left):

$$R'(P) = \frac{æ}{[c - ((æ - 1)/2) f(P)]^2} \left[1 - \frac{(æ - 1) P f'(P)}{c - ((æ - 1)/2) f(P)}\right].$$

To determine P and U, we use the Newton iterative process [13]:

$$\mathbf{x}^{(i)} = \mathbf{x}^{(i-1)} - W^{-1}(\mathbf{x}^{(i-1)}) \mathbf{F}(\mathbf{x}^{(i-1)}),$$

where

$$W^{-1} = \frac{1}{\Delta} \left\| \begin{matrix} F'_{22} & -F'_{12} \\ -F'_{21} & F'_{11} \end{matrix} \right\|, \qquad \Delta = F'_{11} F'_{22} - F'_{12} F'_{21}.$$

As an initial approximation, we should set $P^{(0)} = p_\infty$ and $U^{(0)} = u - f(P^{(0)})$, which corresponds to the condition of full reflection [11].

Implementing the iterative process to convergence, we determine the velocity of the left wave D. If $D < 0$, for "large" values of P, U, and R used in the difference scheme, we assume the values calculated by the proposed procedure. This case (subsonic flow) is most interesting for the problem considered here. In the opposite case ($D \geqslant 0$), we deal with a supersonic flow regime and set $P = p, U = u$, and $R = \rho$.

We note that when the process is described using the equations of acoustics [11], the system corresponding to system (2.3), (2.4) has the form

$$P - p_\infty - \xi \rho_0 \frac{|U|U}{2} = 0, \qquad U - u + \frac{P - p}{\rho_0 c_0} = 0.$$

Excluding the pressure on the discontinuity P from consideration, we arrive at the equation for velocity

$$\frac{\xi}{2c_0} |U|U + U + \frac{p_\infty - p}{\rho_0 c_0} - u = 0.$$

The physically admissible solution has the form

$$U = \frac{(\mp 1 \pm \sqrt{D}) c_0}{\xi}, \qquad D = 1 \mp \frac{2\xi}{c_0} \left(\frac{p_\infty - p}{\rho_0 c_0} - u \right),$$

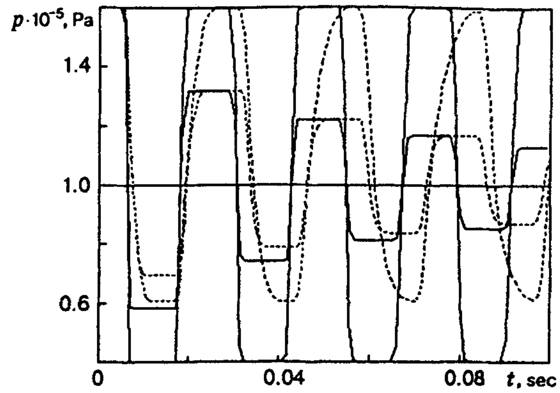

Fig. 1

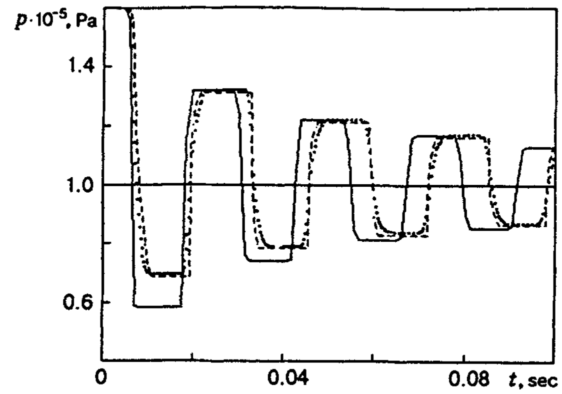

Fig. 2

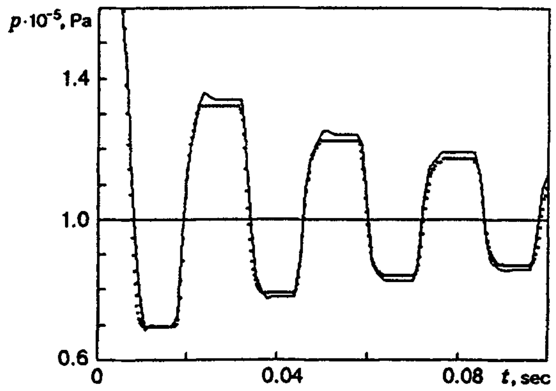

Fig. 3

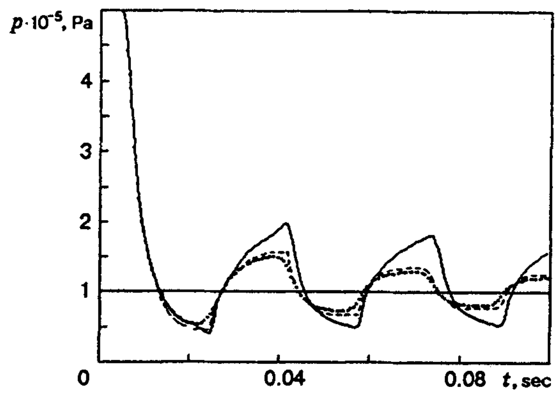

Fig. 4

where the upper signs correspond to outflow and the lower signs correspond to inflow. In both cases, $P = p - (U - u)\rho_0 c_0$.

Obviously, the formulas for calculating the discontinuity decay on the left boundary differ from those given above only by signs and the use of "small" values from the extreme left cell.

3. Analysis of the Calculation Results. A cylindrical pipe of length $L = 2$ m filled with air at pressure $p_0 = 1.6 \cdot 10^5$ Pa was considered. The ambient parameters were $p_\infty = 10^5$ Pa and $\rho_\infty = 1.3$ kg/m^3. The adiabatic exponent is $æ = 1.4$. Unless otherwise specified, the parameter ξ took the value $\xi = 0.5$ when the pipe was evacuated and $\xi = 1.2$ when it was filled. The internal convergence of the solutions was estimated by calculations on a sequence of condensed grids with a number of calculation cells of $J = 25$, 50, and 100. The calculations showed that an admissible solution can be obtained on a rather coarse grid when the calculation domain is divided into 50 calculation intervals ($J = 50$).

Figure 1 shows curves of pressure versus time at the closed end of the pipe with and without allowance for energy losses at the open section. The solid curves are generated by the linearized equations and the dashed curves are obtained invoking Eqs. (1.1) and (1.2). For both models, neglect of the hydrodynamic drag at the outlet section leads to a stable undamped oscillation regime. The curves obtained by the model proposed are characterized by a decrease in the oscillation amplitude, which is in qualitative agreement with experimental results. The difference in the amplitude and period of oscillations between the linear and nonlinear models is due to the different propagation velocities of compression and rarefaction waves.

The numerical results (the solid curve refers to the linear model and the dotted curve to the nonlinear model) and the experimental results of [4] (dashed curve) are shown in Fig. 2. There is good qualitative and quantitative agreement between the experimental results and the results obtained by the nonlinear model (1.1)–(1.4). The linear model is inadequate.

The curves in Fig. 3 are generated by the nonlinear model (1.1)–(1.4) and various numerical methods. The dotted curve corresponds to the Godunov method and the solid curve corresponds to the Lax–Wendroff two-step scheme [14]. In both cases, the boundary condition at the open end of the pipe (1.4) was realized by the procedure described in Sec. 2. Good agreement of the results is observed. The solid curve shows solution oscillations that are typical of high-order accuracy schemes. They are most pronounced when the difference grids are made finer.

Figure 4 shows the results obtained using boundary conditions (1.4) (the dotted curve) and ignoring the hydrodynamic drag (the solid curve) for an initial pressure in the pipe of $p_0 = 5 \cdot 10^5$ Pa. Beginning with $t = 0.02$, the behavior of the curves becomes different. Only the solution with boundary condition (1.4) is in good qualitative and quantitative agreement with the experimental results (the dashed curve) [15].

Thus, in the present work, an effective numerical method is proposed that reduces the non-one-dimensional problem of evacuation (filling) of a pipe to a one-dimensional problem, whose solution agrees with experimental data. This method is best suited to the case where it is required to calculate gas-dynamic processes in a complex pipework, as is done in [5, 6], since direct calculation of the entire gas flow region is difficult.

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